

# Analysis of the Mixing of Solid Particles in A Vertical Cylindrical Mixer Via Discrete Element Method (DEM)

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**Abstract**—The discrete element method (DEM) was employed to investigate the mixing of solid particles in a vertical cylindrical mixer. In order to achieve this goal, simulations are explored on the effects of initial loading (side-side, top-bottom), agitator speed and fill level on the mixing efficiency. Mixing performance with respect to a vertical cylindrical is investigated using the lacey index. The mixing domain was discretized using a 15×15×15 mesh system which was considered as the samples. Simulation results show that particles are broken up constantly when it flowed through the spoiler and chopper. The side-side initial loading provides a better mixing efficiency. With the increase of the agitator speed, the mixing efficiency is accelerated and the mixing time is shortened. The paper describes the total amount of particles affected the mixing efficiency of the mixer and it obtains a better mixing efficiency at the fill level of 55%.

**Index Terms**—Vertical cylindrical mixer, Discrete element method, Solids mixing, Mixing efficiency, Lacey index

## I. INTRODUCTION

Particle mixing is an important step in a variety of applications spanning the ceramic, foods, metallurgical, plastics, and pharmaceuticals industries [1,2]. Despite mixers are commonly used to mix granular materials, the dynamic of powder blending is poorly understood, greatly complicating process design and implementation, as well as delaying the validation of manufacturing processes [3]. Furthermore, while achieving a quality mixture, avoiding particle breakage is also an important factor in mixing processes. The application of numerical simulations is desirable when investigating mixing because they allow for better control of physical properties and a faster analysis. In other words, the numerical simulation can set ideally the physical properties of all particles, although the particles are colored differently [4]. Therefore, numerical simulations play a vital role in the

improvement and optimization of the mixers.

There is a large variety of mixing apparatus and industrial mixers can be broadly classified into the following categories: tumbling mixers, vertical agitated mixers and convective mixers [5]. Many industries are using vertical agitated mixers widely in powder mixing, such as foods, metallurgical, plastics, and pharmaceuticals. In vertical agitated mixers, particles are mixed mechanically by a rotating blade inside a stationary vessel. The advantages of the mixers are flexible arrangement, simple mixing mechanisms, convenient manipulation and maintenance. Besides, the vertical agitated mixers are applicable to dry and flowing materials [6,7]. However, there are some negative elements on high speed mixer. One of them is that these mixers are not suitable for very cohesive powder. Moreover, the high-speed rotating blade and chopper may damage the larger particles [8].

## II. GEOMETRIC MODEL

The structure of the vertical cylindrical mixer is shown in Fig.1.

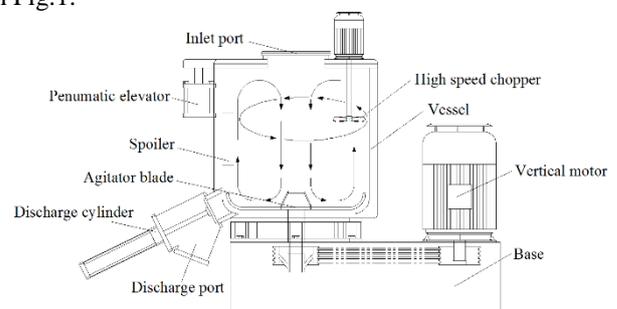


Figure 1. The operational principle of the vertical cylindrical mixer.

The agitator blade let particles move tangentially by apparent friction and side thrust. Then, particles are thrown to the interior wall of the vessel due to the centrifugal force. Besides, particles move upward along the wall. When elevated to a certain height, particles move back to the center of the blade because of the gravity. Then it is

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thrown up again. The combination between upward movement and the tangential movement makes the material under a continuous spiral up and down condition.

As is shown in Fig.1, the computational model for numerical simulation consisted of agitator blade, high-speed chopper, spoiler and vessel.

### III. THE DISCRETE ELEMENT METHOD

#### A. Solving method

Discrete element method (DEM) is a numerical method for simulating discrete matter in a series of events called time-step. By generating particles and controlling the interaction between them using contact models, the forces acting on all particles can be calculated [9]. Newton's second law of motion is then applied and the position of all particles can be calculated for next time-step. When this is repeated it gives the capability of simulating how particles are flowing in particle-machine systems, as is seen in Fig.2. It is also possible to apply external force fields in order to simulate the influence of e.g. air drag or electrostatics. By importing CAD (Computer aided design) geometry and setting dynamic properties the environment which the rock particles is subjected to can be emulated in a very precise manner. This gives full control over most of the parameters and factors that are active and interesting during a crushing sequence. Also, due to the fact that all particles positions, velocities and forces are stored in every time-step, it is possible to observe particle trajectories and flow characteristics.

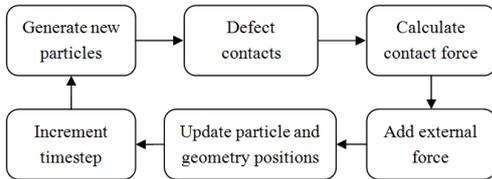


Figure 2.Illustration of the DEM calculation loop used in EDEM.

Tab.1 lists the parameters (properties of acrylic bead and stainless steel) used in this study for DEM simulations.

TABLE I.  
LIST OF PARAMETERS USED IN THE DEM SIMULATION

Materials	Poisson ratio	0.35	
	Density/(kg·m <sup>-3</sup> )	1200	
	Shear modulus/Pa	8.9×10 <sup>8</sup>	
Interactions	Type	particle-part icle	particle-wall
	Model	Hertz-Mind lin(no-slip)	Hertz-Mindli n(no-slip)
	Coefficient of Restitution	0.45	0.45
	Coefficient of Static Friction	0.5	0.5
	Coefficient of Rolling Friction	0.05	0.01

#### B. Hertz-Mindlin Contact model

The Hertz-Mindlin contact model, Fig.3 is used for accurately calculating forces for particles-particle and

particles-geometry interactions in the simulation. The normal force component is derived Hertzian contact theory and the tangential component from work done by Mindlin [10].

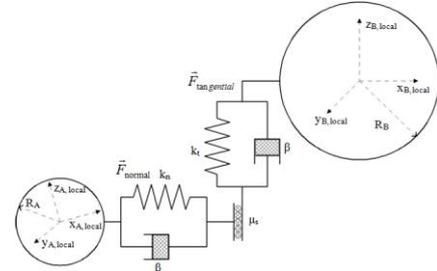


Figure 3.Schematic illustration of the Hertz-Mindlin contact model used in EDEM.

Damping components are added to normal and tangential force components where damping coefficients are linked to coefficient of restitution. The normal force is given by considering the normal overlap  $U_n$  according to:

$$\vec{F}_{normal} = \frac{4}{3} E^* \sqrt{R^*} U_n^{3/2}. \quad (1)$$

The damping force is given by:

$$\vec{F}_{normal}^d = -2\sqrt{5/6}\beta\sqrt{k_n m^*} v_n^{rel}. \quad (2)$$

Here,  $E^*$  is the equivalent Young's modulus,  $R^*$  is the equipment radius,  $m^*$  is the equivalent mass,  $\beta$  is the damping coefficient,  $k_n$  is the stiffness,  $v_n^{rel}$  is the relative normal velocity and is given by:

$$\frac{1}{E^*} = \frac{1}{E_i} \frac{\mu_i^2}{E_i} + \frac{1}{E_j} \frac{\mu_j^2}{E_j}. \quad (3)$$

Here,  $E_i$  and  $E_j$  are the Young's modulus for spheres in contact,  $\mu_i$  and  $\mu_j$  are the Poisson ratio spheres in contact.

$$\frac{1}{R^*} = \frac{1}{R_i} + \frac{1}{R_j}. \quad (4)$$

Here,  $R_i$  and  $R_j$  are the radius for spheres in contact.

$$m^* = \left( \frac{1}{m_i} + \frac{1}{m_j} \right). \quad (5)$$

$$\beta = \frac{\ln e}{\sqrt{\ln^2 e + \pi^2}}. \quad (6)$$

$$k_n = 2E^* \sqrt{R^*} U_n. \quad (7)$$

The tangential force component is defined as the tangential stiffness times the tangential overlap. In addition the tangential damping force and tangential stiffness given by:

$$\vec{F}_{tangential} = k_t U_t. \quad (8)$$

Here,  $U_t$  is the tangential overlap.

$$\vec{F}_{\text{tangential}}^d = -2\sqrt{5/6}\beta\sqrt{k_t m^*} \vec{v}_t \quad (9)$$

Here,  $k_t$  is the tangential stiffness,  $v_{rel t}$  is the relative tangential velocity.  $k_t$  is given by:

$$k_t = 8G^* \sqrt{R^* U_n} \quad (10)$$

C. *Mixing index*

Lacey's mixing index  $M$ , which has been widely employed for various mixers [11], is used to evaluate the degree of mixing and is expressed as:

$$M = \frac{\sigma_0^2 - \sigma_t^2}{\sigma_0^2 - \sigma_r^2} \quad (11)$$

Here  $\sigma_t^2$  is the variance of the mixture.

$$\sigma_t^2 = \frac{1}{p-1} \sum_{i=1}^p (c_i - c_m)^2 \quad (12)$$

Here,  $\sigma_0^2$  is the variance in a completely segregated system.

$$\sigma_0^2 = c_m(1 - c_m) \quad (12)$$

And  $\sigma_r^2$  is the variance in a perfectly mixed system.

$$\sigma_r^2 = \frac{c_m(1 - c_m)}{q} \quad (13)$$

Where  $p$  is total number of sampling cells from this system,  $c_i$  is the local concentration in a sampling cell,  $c_m$  is the overall proportion of one type of particle in the system, and  $q$  is the average particle number over all sampling cells.

Number of sampling cells or cell size might influence estimation of the degree of mixing. In order to measure the lacey index (variance of the mixture), mixing domain is discretized using a  $15 \times 15 \times 15$  mesh system (as is seen in Fig.4). Each mesh contains approximately 55-65 particles which is consistent with the two colors of acrylic beads. In the calculation of variance, if the total number of the particles present in a sample was below the average number of particles in all the samples, that sample was eliminated from the calculations. The reason is that the small number of particles in cells can inaccurately change the variance by a large number [12].

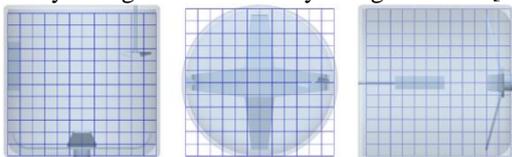


Figure 4. Schematic of the vertical cylindrical mixer with cubic cells.

IV. RESULTS AND DISCUSSION

A. *Effect of initial loading*

The effect of initial loading (side-side, top-bottom) on

the mixing rate of the vertical cylindrical mixer was explored. To achieve this goal, 20,000 red acrylic beads (4mm diameter) were mixed with 20,000 black acrylic beads (4mm diameter) to achieve the 55% fill level of the mixer's geometry. Simulations lasted for 2s and the agitator blade was rotating at 1,000 rpm. Moreover, the angular velocity of the high-speed chopper was 10,000 rpm and the directions of the rotation of the agitator and the chopper were the same.

Fig.5 shows the mixing index versus time for both initial loadings. It can be seen that the side-side loading provided a more efficient than top-bottom loading. According to these data, the mixing index are the same after 1.0s and reach a plateau for both loading. Besides, the lacey index for the top-bottom loading is higher than side-side loading at zero seconds. The reason for the higher lacey index in the top-bottom case was that the initial surface area between the red and black particles was higher than that for side-side loading.

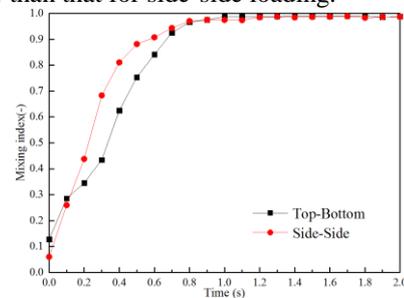


Figure 5. Mixing index versus time for different initial loading at the fill level of 55% and the blade speed of 1000 rpm.

Although the mixing index is higher for the top-bottom loading at the beginning, it presented a faster growth trend for the side-side loading. Reasons that resulted in such a phenomenon are the mixing of particles in vertical cylindrical mixer is a cyclic progress. Under the top-bottom condition, the upper red particles moved to the bottom of the vessel to mix the black particles firstly. But both of the two kinds of particles are at the bottom, it reduced the mixing time. In the other words, since bulk convection was stronger in side-side initial loading, the slope of the corresponding graph was bigger.

Fig.6 depicts illustrates the snapshots of the vertical cylindrical mixer for the two initial loadings at different times of the mixing process. As is seen in Fig.6, at the beginning, the red and black particles were completely segregated. As the time passed, the acrylic bead particles started to mix gradually, and at the end of the simulation, a solid mixture with a high mixing quality was achieved.

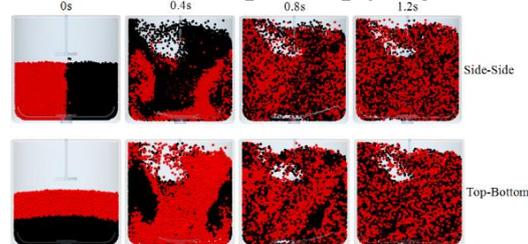


Figure 6. Snapshots of the simulated solid mixture at times equal to 0s, 0.4s, 0.8s, 1.2s and a rotational speed of 1000 rpm for 55% fill level.

**B. Effect of agitator speed**

The speed of rotating blade affects the granular flow in a high speed mixer. In order to achieve an acceptable particle movement or shear rate, low rotation speed have been avoided. In addition, a high rotational speed may pick the particles up and throw them into the high space of the mixer. It can make most use of the space and reduce the possibility of particle breakage. In order to obtain the effect of the rotational speed, the simulations were conducted at three different rotational speeds (800, 1000 and 1200 rpm). The fill level of 55% with top-bottom loading of 20,000 red acrylic beads (4mm diameter) and 20,000 black acrylic beads (4mm diameter). Moreover, the angular velocity of the high-speed chopper was 10,000 rpm and the directions of the rotation of the agitator and the chopper were the same.

Fig.7 shows the mixing index versus time at different blade speeds. Although the lacey index reached a plateau at the steady state, the maximum is achieved faster at the higher rotational speed. Because the higher speed blade could provide more energy for the particles and the particles would obtain higher speed and acceleration in such a situation. Therefore, with increasing the blade rotational speed, the number of mixing cycles per unit time was increased resulting in a higher mixing index.

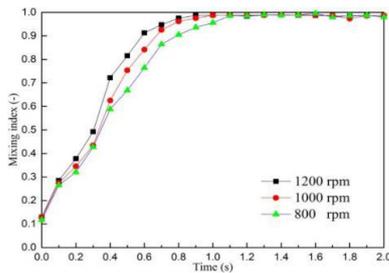


Figure 7. Mixing index versus time at different blade speed, the fill level of 55%, and top-bottom initial loading.

Fig.8 shows the velocity of the particles in vertical cylindrical mixer while the blade speed was 1000 rpm. As can be seen, the particles near the blade have the highest velocity. This can be explained by Newton’s second law of motion. The agitator provided more energy to the particles near the blade than the other particles. So it provides higher speed and acceleration to particles.

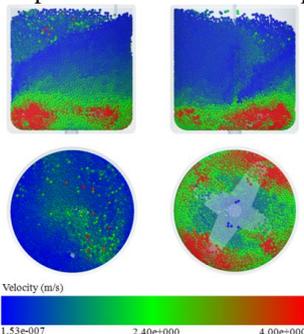


Figure 8. Velocities of the particles in the high speed mixer at fill level of 55%, the top-bottom initial loading, and the agitator blade speed of 1000 rpm.

**C. Effect of the fill level**

The fill level of the mixer in an important factor in industry, mainly because it often improved the structure of the vertical cylindrical mixer based on the maximum fill level. In order to obtain the effect of fill levels on performance of the high speed mixer, simulations were performed for 45%, 55% and 65% (as is seen Tab.2). Furthermore, the initial loading method was the top-bottom with the particle diameter of 4 mm and the total number of particles was divided into two halves of two colors. The angular velocity of the high-speed chopper was 10,000 rpm and the directions of the rotation of the agitator and the chopper were the same.

TABLE II.  
SIMULATION PARAMETERS

Fill level(%)	Particle size (diameter, mm)	Number of particles (total)	Agitator blade speed (rpm)
45	4	30000	1000
55	4	40000	1000
65	4	50000	1000

Fig.9 shows the lacey index versus time at different fill levels. According to these data, the vertical cylindrical mixer provides a better performance at the 55% fill level compared to those achieved other two fill levels. It can be seen that the vertical cylindrical mixer provides a low mixing rate at the 65% fill level. This is due to the fact that particles had low velocity and less space to move at the higher fill levels. On the other hand, the low fill lever obtains a higher mixing index before 0.4 seconds. But it becomes lower than the 55% fill level after 0.4 seconds. The reason behind this phenomenon might be that the solid particles obtain a high velocity at the bottom of the mixer. But the solid particles, near the walls, are held on the walls due to the centrifugal force. It reduces the recycle rate of the particles.

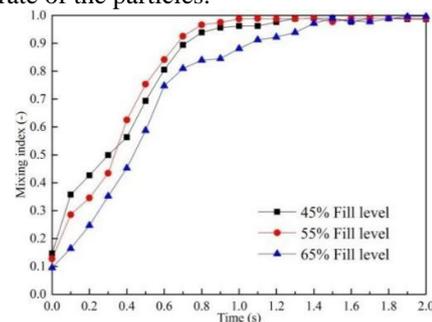


Figure 9. Mixing index versus time at different fill levels, the blade speed of 1000 rpm, and the top-bottom initial loading.

**V. CONCLUSION**

This paper focused on the computer simulations of the granular mixing via DEM. The numerical simulations clarified the mixing mechanism behind observed phenomenon. The effects of initial loadings, blade speeds and fill levels on the degree of mixing were investigated. The following conclusions can be obtained:

(1) The high-speed moving particles formed a vortex due to the centrifugal force. It slowed down when the

particles moved to the spoiler. And the flowing state was changed. In addition, the particles' aggregation was reduced when it flow past the high-speed chopper.

(2) In this paper, for both side-side and top-bottom loadings, the mixing indices were the same after a while. But the side-side loading provided a more efficient mixing rate than top-bottom loading.

(3) An analysis of the simulation data shows that, for the analyzed vertical cylindrical mixer, the blade speed are an important factor that affect the mixing quality. The mixing time reduced with an increase in the rotational speed.

(4) The amount of powder is a more important factor, without a sufficient fill level, a good mixture cannot be achieved even at high mixing speeds. The vertical cylindrical mixer provided a better performance at 55% fill level compared to other fill levels.

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